BC 2

Sample Examination II

Section I Part A

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given. Do not spend too much time on any one problem. Calculators may NOT be used on this part of the exam.

In this test: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. $\int_0^2 (2x^3 + 3)\, dx =$

(A) 8
(B) 11
(C) 14
(D) 20
(E) 24

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2. In decomposing $\frac{5x-2}{(x-7)(x+4)}$ by the method of partial fractions, one of the fractions obtained is

(A) $\frac{-2}{x-7}$
(B) $\frac{2}{x-7}$
(C) $\frac{3}{x-7}$
(D) $\frac{3}{x+4}$
(E) $\frac{5}{x+4}$

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3. A particle moves in the xy-plane so that at any time t its coordinates are $x = t^2$ and $y = 4 - t^3$. At $t = 1$, its acceleration vector is

(A) $(2, -3)$
(B) $(2, -6)$
(C) $(1, -6)$
(D) $(2, 6)$
(E) $(1, -2)$

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4. If $f(x) = (2 + 3x)^4$, then the fourth derivative of f is

(A) 0
(B) $4!(3)$
(C) $4!(3^4)$
(D) $4!(3^5)$
(E) $4!(2 + 3x)$

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5. At what value(s) of x does $f(x) = x^4 - 8x^2$ have a relative minimum?

(A) 0 and −2 only
(B) 0 and 2 only
(C) 0 only
(D) −2 and 2 only
(E) −2, 0, and 2

6. $\int \sqrt{x}(x+2)\,dx =$

(A) $\frac{2}{5}x^{\frac{5}{2}} + \frac{4}{3}x^{\frac{3}{2}} + C$ (B) $\frac{2}{5}x^{\frac{3}{2}} + \frac{4}{3}x^{\frac{1}{2}} + C$ (C) $\frac{3}{2}\sqrt{x} + \frac{1}{\sqrt{x}} + C$

(D) $\sqrt{x^3} + 2\sqrt{x} + C$ (E) $\frac{x^2}{2}\left(\frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}}\right) + C$

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7. A curve in the xy-plane is defined parametrically by the equations $x = t^2 + t$ and $y = t^2 - t$. For what value of t is the tangent line to the curve horizontal?

(A) $t = -1$

(B) $t = -\frac{1}{2}$

(C) $t = 0$

(D) $t = \frac{1}{2}$

(E) $t = 1$

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8. The function $y = x^4 + bx^2 + 8x + 1$ has a horizontal tangent and a point of inflection for the same value of x. What must be the value of b?

(A) -6

(B) -1

(C) 1

(D) 4

(E) 6

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9. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = y - x$. The point $(5, 1)$ is on the graph of the solution to this differential equation. What is the approximation for $f(6)$ if Euler's Method is used, starting at $x = 5$ with a step size of 0.5?

(A) -4.25

(B) -3.25

(C) -1.25

(D) 0.75

(E) 2.25

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11. $\lim_{x\to 2} \frac{x^2 - 4}{\int_2^x \cos(\pi t)\,dt}$ is

(A) 0 (B) 1 (C) 2

(D) 4 (E) undefined

12. Which of the following improper integrals converge?

I. $\int_0^{\infty} e^{-x}\,dx$

II. $\int_0^{1} \frac{1}{x^2}\,dx$

III. $\int_0^{1} \frac{1}{\sqrt{x}}\,dx$

(A) I only

(B) III only

(C) I and II only

(D) II and III only

(E) I and III only

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13. If $x + y = xy$, then $\frac{dy}{dx}$ is

(A) $\frac{1}{x-1}$

(B) $\frac{1-y}{x-1}$

(C) $\frac{y-1}{x-1}$

(D) $x + y - 1$

(E) $\frac{2-xy}{y}$

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14. If f and g are continuously differentiable functions defined for all real numbers, which of the following definite integrals is equal to $f(g(4)) - f(g(2))$?

(A) $\int_2^4 f'(g(x))\,dx$

(B) $\int_2^4 f(g(x))f'(x)\,dx$

(C) $\int_2^4 f(g(x))g'(x)\,dx$

(D) $\int_2^4 f(g'(x))g'(x)\,dx$

(E) $\int_2^4 f'(g(x))g'(x)\,dx$

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15. The velocity of a particle moving along the y-axis is given by $v(t) = 8 - 2t$ for $t \geq 0$. The particle moves upward until it reaches the origin and then moves downward. The position of the particle at any time t is given by

(A) $-t^2 + 8t - 16$

(B) $-t^2 + 8t + 16$

(C) $2t^2 - 8t - 16$

(D) $8t - 2t^2$

(E) $8t - t^2$

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16. If the substitution $u = \sqrt{x-1}$ is made, the integral $\int_2^5 \frac{\sqrt{x-1}}{x}\,dx =$

(A) $\int_2^5 \frac{2u^2}{u^2+1}\,du$

(B) $\int_1^2 \frac{u}{u^2+1}\,du$

(C) $\int_1^2 \frac{u^2}{2(u^2+1)}\,du$

(D) $\int_2^5 \frac{u}{u^2+1}\,du$

(E) $\int_1^2 \frac{2u^2}{u^2+1}\,du$

18. If $\int_0^2 (2x^3 - kx^2 + 2k)\, dx = 12$, then k must be

(A) -3

(B) -2

(C) 1

(D) 2

(E) 3

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20. For $|x| < 1$, the derivative of $y = \ln\sqrt{1 - x^2}$ is

(A) $\frac{x}{1-x^2}$ (B) $\frac{-x}{1-x^2}$ (C) $\frac{-x^2}{x^2-1}$ (D) $\frac{1}{2(1-x^2)}$ (E) $\frac{1}{\sqrt{1-x^2}}$

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21. A function whose derivative is a constant multiple of the function itself must be

(A) periodic

(B) linear

(C) logarithmic

(D) quadratic

(E) exponential

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22. What are all values of x for which the graph of $y = x^3 - 6x^2$ is concave downward?

(A) $0 < x < 4$

(B) $x < 2$

(C) $x > 2$

(D) $x < 0$

(E) $x > 4$

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23. The rate of decay of a radioactive substance is proportional to the amount of substance present. Four years ago there were 12 grams of the substance. Now there are 8 grams. How many grams will there be 8 years from now?

(A) 0

(B) $\frac{8}{3}$

(C) $\frac{32}{9}$

(D) $\frac{81}{16}$

(E) $\frac{16}{3}$

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26. A normal line to the graph of a function f at the point $(x, f(x))$ is defined to be the line perpendicular to the tangent line at that point. An equation of the normal line to the curve $y = \sqrt[3]{x^2 - 1}$ at the point where $x = 3$ is

(A) $y + 12x = 38$

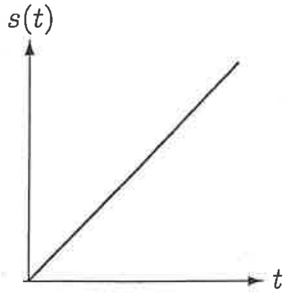
(B) $y - 4x = 10$

(C) $y + 2x = 4$

(D) $y + 2x = 8$

(E) $y - 2x = -4$

27. Which graph best represents the position of a particle, $s(t)$, as a function of time, if the particle's velocity and acceleration are both positive?

(A)


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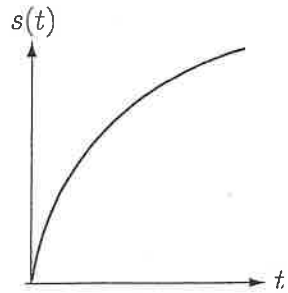
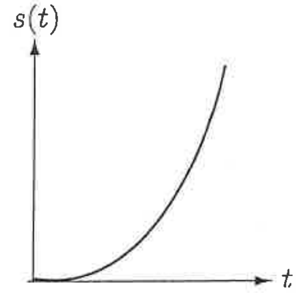


(C)


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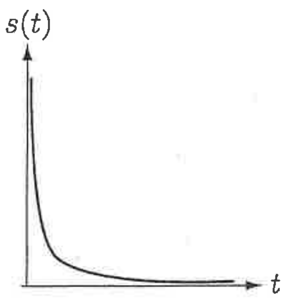
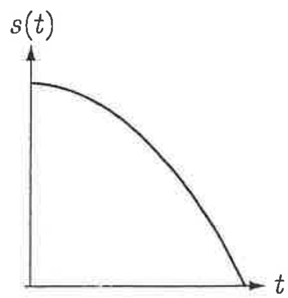


(E)


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28. If n is a positive integer, then $\lim_{n\to\infty} \frac{1}{n}\left[\left(\frac{1}{n}\right)^2 + \left(\frac{2}{n}\right)^2 + \cdots + \left(\frac{n-1}{n}\right)^2\right] =$

(A) $\int_0^1 \frac{1}{x^2}\,dx$

(B) $\int_0^1 x^2\,dx$

(C) $\int_0^1 \frac{2}{x^2}\,dx$

(D) $\int_0^1 \frac{1}{x}\,dx$

(E) $\int_0^2 x^2\,dx$

Section I Part B

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given. Do not spend too much time on any one problem. A graphing calculator is required for some questions on this part of the examination.

In this test:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

29. If the position of a particle moving in the xy-plane is given by the parametric equations $x(t) = 9\cos t$ and $y(t) = 4\sin t$ for $t \geq 0$, then at $t = 3$, the acceleration vector is

(A) $(-8.910, 0.564)$

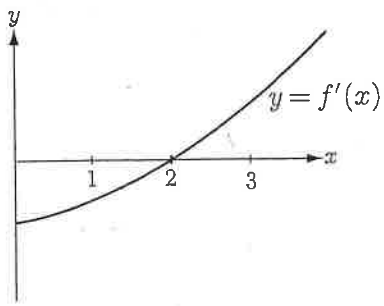
(B) $(-1.270, -3.960)$

(C) $(8.910, -0.564)$

(D) $(8.910, 0.564)$

(E) $(-0.564, 8.910)$

30.



The graph of the derivative of a twice-differentiable function f is shown above. If $f(1) = -2$, which of the following is true?

(A) $f(2) < f'(2) < f''(2)$

(B) $f''(2) < f'(2) < f(2)$

(C) $f'(2) < f(2) < f''(2)$

(D) $f(2) < f''(2) < f'(2)$

(E) $f'(2) < f''(2) < f(2)$

31. Let f be a function that is everywhere differentiable. The value of $f'(x)$ is given for several values of x in the table below.

x	–10	–5	0	5	10
$f'(x)$	–2	–1	0	1	2

If $f'(x)$ is always increasing, which statement about $f(x)$ must be true?

(A) $f(x)$ has a relative minimum at $x = 0$.

(B) $f(x)$ is concave downwards for all x.

(C) $f(x)$ has a point of inflection at $(0, f(0))$.

(D) $f(x)$ passes through the origin.

(E) $f(x)$ is an odd function.

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32. A certain species of fish will grow from x million to $x(15 - x)$ million each year. In order to sustain a steady catch each year, a limit of $x(15 - x) - x$ million fish are to be caught, leaving x million fish to reproduce each year. What is the number of fish which should be left to reproduce each year so that the maximum catch may be sustained from year to year?

(A) 5 million

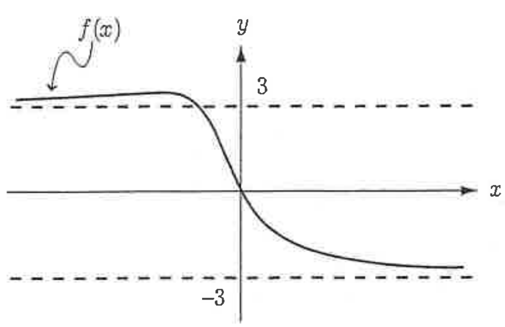
(B) 7 million

(C) 7.5 million

(D) 10 million

(E) 15 million

33.



The figure above shows the graph of a function $f(x)$ which has horizontal asymptotes of $y = 3$ and $y = -3$. Which of the following statements are true?

I. $f'(x) < 0$ for all $x \geq 0$

II. $\lim_{x \to +\infty} f'(x) = 0$

III. $\lim_{x \to -\infty} f'(x) = 3$

(A) I only

(B) II only

(C) III only

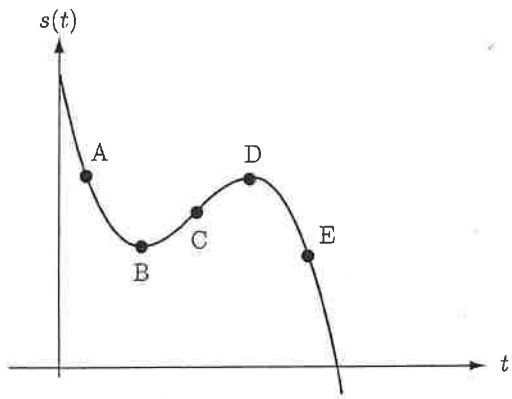
(D) I and II only

(E) I, II, and III

34. The velocity vector of a particle moving in the coordinate plane is $(4t, -2t)$ for $t \geq 0$. The path of the particle lies on

(A) a hyperbola (B) an ellipse (C) a line (D) a parabola (E) a ray

35.



The graph above shows the distance $s(t)$ from a reference point of a particle moving on a number line, as a function of time. Which of the points marked is closest to the point where the acceleration first becomes negative?

(A) A
(B) B
(C) C
(D) D
(E) E

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36. The derivative of f is given by $f'(x) = e^x(-x^3 + 3x) - 3$, for $0 \leq x \leq 5$.

At what value of x does $f(x)$ attain its absolute minimum?

(A) For no value of x
(B) 0
(C) 0.618
(D) 1.623
(E) 5

37.

x	$f(x)$
3.99800	1.15315
3.99900	1.15548
4.00000	1.15782
4.00100	1.16016
4.00200	1.16250

The table above gives values of a differentiable function f. What is the approximate value of $f'(4)$?

(A) 0.00234

(B) 0.289

(C) 0.427

(D) 2.340

(E) 4.270

38. If $y = 7$ is a horizontal asymptote of a rational function f, then which of the following must be true?

(A) $\lim_{x \to 7} f(x) = \infty$

(B) $\lim_{x \to \infty} f(x) = 7$

(C) $\lim_{x \to 0} f(x) = 7$

(D) $\lim_{x \to 7} f(x) = 0$

(E) $\lim_{x \to -\infty} f(x) = -7$

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39. In the interval $0 \leq x \leq 5$ the graphs of $y = \cos 2x$ and $y = \sin 3x$ intersect four times. Let A, B, C, and D be the x-coordinates of these points so that $0 < A < B < C < D < 5$. Which of the definite integrals below represents the largest number?

(A) $\int_0^A (\cos 2x - \sin 3x)\ dx$

(B) $\int_A^B (\sin 3x - \cos 2x)\ dx$

(C) $\int_B^C (\sin 3x - \cos 2x)\ dx$

(D) $\int_C^D (\sin 3x - \cos 2x)\ dx$

(E) $\int_C^D (\cos 2x - \sin 3x)\ dx$

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40. The function $f(x) = \tan(3^x)$ has one zero in the interval $[0, 1.4]$. The derivative at this point is

(A) 0.411

(B) 1.042

(C) 3.451

(D) 3.763

(E) undefined

41.

x	0	1	2	3	4	5	6
$f(x)$	0	0.25	0.48	0.68	0.84	0.95	1

For the values of a continuous function given in the table above, $\int_0^6 f(x)\ dx$ is approximated by a Riemann sum using the value at the midpoint of each of three intervals of width 2. The approximation is

(A) 2.64

(B) 3.64

(C) 3.72

(D) 3.76

(E) 4.64

42. $\frac{d}{dx}\int_{x}^{x^3} \sin(t^2)\, dt =$

(A) $\sin(x^6) - \sin(x^2)$

(B) $6x^2 \sin(x^3) - 2\sin x$

(C) $3x^2 \sin(x^6) - \sin(x^2)$

(D) $6x^5 \sin(x^6) - 2\sin(x^2)$

(E) $2x^3 \cos(x^6) - 2x\cos(x^2)$

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43. A tank is being filled with water at the rate of $300\sqrt{t}$ gallons per hour with $t > 0$ measured in hours. If the tank is originally empty, how many gallons of water are in the tank after 4 hours?

(A) 600

(B) 900

(C) 1200

(D) 1600

(E) 2400

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44. The region in the first quadrant enclosed by the graphs of $y = x$ and $y = 2\sin x$ is revolved about the x-axis. The volume of the solid generated is

(A) 1.895

(B) 2.126

(C) 5.811

(D) 6.678

(E) 13.355

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45. The tangent line to the graph $y = e^{2-x}$ at the point $(1, e)$ intersects both coordinate axes. What is the area of the triangle formed by this tangent line and the coordinate axes?

(A) $2e$

(B) $e^2 - 1$

(C) e^2

(D) $2e\sqrt{e}$

(E) $4e$